Computational details for : ”Optimal Estimation of the Centroidal Dynamics of Legged Robots”

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This document complements the paper entitled "Differential Dynamic Programming for Maximum a Posteriori Centroidal State Estimation of Legged Robots" [1]. The purpose of this work was to estimate the centroidal dynamics of legged robots by formulating a maximum a posteriori problem and solving it thanks to differential dynamic programming (DDP). In the following, the computations of the partial derivatives of the unoptimized value function ($\mathcal{Q}_k$) are provided for the DDP algorithm. Then, the hypothesis about the $0-$mean property of the stochastic part of the dynamics is validated by Fig. 1 (result of the simulation) which demonstrates that the DDP minimization of Eq.(11) does keep ω_k $0-$mean.

Appendix I
Partial derivatives of Q_k

- $Q_{xk} = \nabla_{\mathbf{x_k}} l_k + \nabla_{\mathbf{x_k}} \mathcal{V}_{k+1}(f(\mathbf{x_k}, \boldsymbol{\omega_k}))$,

$$\nabla_{\mathbf{x_k}} l_k = 2\frac{\partial(g(\mathbf{x_k}) - \mathbf{y_k})^T}{\partial \mathbf{x_k}} \Sigma_{\boldsymbol{\eta_k}}^{-1}(g(\mathbf{x_k}) - \mathbf{y_k}),$$
$$\frac{\partial g(\mathbf{x_k})}{\partial \mathbf{x_k}} = \frac{C(\mathbf{x_k})}{\partial \mathbf{x_k}}\mathbf{x_k} + C(\mathbf{x_k}) = \begin{bmatrix} \mathbb{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\mathbf{c}_k\times & 0 & \mathbb{1} \\ 0 & 0 & \mathbb{1} & 0 & 0 \\ 0 & 0 & 0 & \mathbb{1} & 0 \end{bmatrix} \hat{=} \tilde{C}(\mathbf{x_k}),$$
$$Q_{xk} = 2\tilde{C}(\mathbf{x_k})^T \Sigma_{\boldsymbol{\eta_k}}^{-1}(g(\mathbf{x_k}) - \mathbf{y_k}) + A^T \mathcal{V}'_x.$$

- $Q_{\omega k} = \nabla_{\boldsymbol{\omega_k}} l_k + \nabla_{\boldsymbol{\omega_k}} \mathcal{V}_{i+1}(f(\mathbf{x_k}, \boldsymbol{\omega_k}))$,

$$\nabla_{\boldsymbol{\omega_k}} l_k = \frac{\partial ||\boldsymbol{\omega_k}||^2_{\Sigma^{-1}_{\boldsymbol{\omega_k}}}}{\partial \boldsymbol{\omega_i}} = 2\Sigma_{\boldsymbol{\omega_k}}^{-1}\boldsymbol{\omega_k},$$
$$Q_{\omega k} = 2\Sigma_{\boldsymbol{\omega_k}}^{-1}\boldsymbol{\omega_k} + B^T \mathcal{V}'_x.$$

- $Q_{xxk} = \nabla^2_{\mathbf{x_k}} l_k + \nabla^2_{\mathbf{x_k}} \mathcal{V}_{i+1}(f(\mathbf{x_k}, \boldsymbol{\omega_k}))$,

where, the element of $\nabla^2_{\mathbf{x_k}} l_k$ at the i^{th} row and j^{th} column is denoted by:

$$[\nabla^2_{\mathbf{x_k}} l_k]_{ij} = \sum_{m=1}^{Y}\sum_{n=1}^{Y}[\Sigma_{\boldsymbol{\eta_k}}^{-1}]_{mn}\left(\frac{\partial \tilde{C}^T_{im}}{\partial \mathbf{x}_j}[g(\mathbf{x_k}) - \mathbf{y_k}]_n + \tilde{C}^T_{im}\frac{\partial [g(\mathbf{x_k})]_n}{\partial \mathbf{x}_j}\right),$$
$$[\nabla^2_{\mathbf{x_k}} l_k]_{ij} = [\tilde{C}^T\Sigma_{\boldsymbol{\eta_k}}^{-1}\tilde{C}]_{ij} + \sum_{n=1}^{Y}\sum_{m=1}^{Y}\frac{\partial \tilde{C}^T_{km}}{\partial \mathbf{x}_l}[\Sigma_{\boldsymbol{\eta_k}}^{-1}]_{mn}[g(\mathbf{x_k}) - \mathbf{y_k}]_n,$$
$$[\nabla^2_{\mathbf{x_k}} l_k]_{ij} = [\tilde{C}^T\Sigma_{\boldsymbol{\eta_k}}^{-1}\tilde{C}]_{ij} + \tilde{\tilde{\mathbf{c}}}^T_{ij}\Sigma_{\boldsymbol{\eta_k}}^{-1}(g(\mathbf{x_k}) - \mathbf{y_k}),$$

where, $\tilde{\tilde{\mathbf{c}}}_{ij} \in \mathbb{R}^Y s$ is the stacked vector of $\frac{\partial \tilde{C}^T_{im}}{\partial \mathbf{x}_j}$ for $m \in [1..Y]$.

- $Q_{\omega\omega k} = \nabla^2_{\boldsymbol{\omega_k}} l_k + \nabla^2_{\boldsymbol{\omega_k}} \mathcal{V}_{i+1}(f(\mathbf{x_k}\boldsymbol{\omega_k}))$,

$$\nabla^2_{\boldsymbol{\omega_k}} l_k = 2\Sigma_{\boldsymbol{\omega_k}}^{-1}\frac{\partial \boldsymbol{\omega_k}}{\partial \boldsymbol{\omega_k}} = 2\Sigma_{\boldsymbol{\omega_k}}^{-1}$$
$$Q_{\omega\omega k} = 2\Sigma_{\boldsymbol{\omega_k}}^{-1} + B^T \mathcal{V}_{xx} B$$

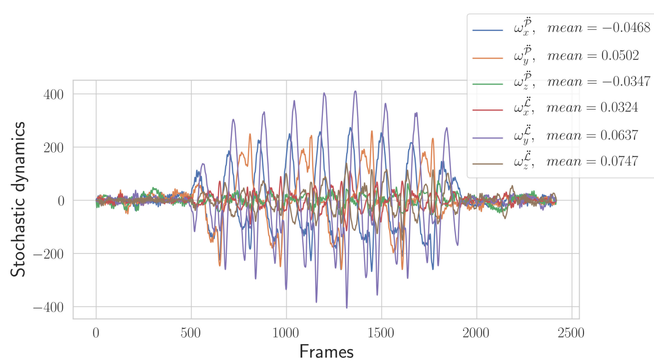

Fig. 1: Time evolution of the stochastic control inputs of the dynamics for the simulated walk of the HRP-2 robot.

References

[1] F. Bailly, J. Carpentier, and P. Souères, "Optimal estimation of the centroidal dynamics of legged robots," in *Internat. Conf. on Robotics and Automation*. IEEE, 2021.

[a] Laboratoire de Simulation et Modélisation du Mouvement, Faculté de Médecine, Université de Montréal, Laval, QC, CanadaLAAS-CNRS, 7 Avenue du Colonel Roche, F-31400 Toulouse, France

[b] LAAS-CNRS, 7 Avenue du Colonel Roche, F-31400 Toulouse, France

[c] Inria, Département d'informatique de l'ENS, École normale supérieure, CNRS, PSL Research University, Paris, France

[*]corresponding author: francois.bailly@umontreal.ca